Mathematical modeling of thin layer microwave drying kinetics of elephant foot yam (*Amorphophallus paeonifolius*)


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**Abstract**

The effect of microwave power ranging from 180 W to 900 W was studied on elephant foot yam. The various factors of the sample that were investigated were drying time, drying rate, kinetic rate constant, effective moisture diffusivity. It was observed that the drying rate was increasing with increase in microwave power and drying time decrease with increase in microwave power. Various thin layer drying models were considered for the evaluation of parameters of drying kinetics, out of which semi empirical Midilli *et al.* model presented the best fit for all conditions of drying, giving a high value of $R^2$ (> 0.998) and low value of RSS, RMSE and Chi square. The Fick’s diffusion method was also used to estimate the effective moisture diffusivity and values were found to be in range of $4.44 \times 10^{-9}$ m$^2$/s to $4.2 \times 10^{-8}$ m$^2$/s at the drying conditions. The modified Arrhenius type equation was used to calculate the activation and the resulting range was from 24.7 Wg$^{-1}$ for varying thickness of the sample.

**Keywords**

Elephant foot yam, Microwave drying, Microwave power, Effective moisture diffusivity, Activation energy

**Introduction**

The Elephant foot yam or White-spot giant arum which is scientifically known as *Amorphophallus paeonifolius* has its origin from the south East Asia and comes from the family Araceae (Hedrick, 1972) and sub-family Aroideae. It is rich in starch and various proteins. It is characterized as a tuber and has wide uses in Ayurvedic medicine (Angayarkanni *et al.*, 2007) including its roots which are used to treat boils and opthalmia. Various reports and studies suggest that it also has antioxidant, synergistic, anti-depressant, antibacterial, antifungal and cytotoxic activity, thermogenic, analgesic, anthelmintic, liver tonic, diuretic and also for treating piles, dyspnoea, spleenomegaly and cough (Hurkadale *et al.*, 2012). It is also used as a staple food in several countries like Bangladesh, Indonesia, Sumatra, Malaysia, Philippines, Java, Srilanka, India, China and south eastern Asian countries (Chandra, 1984; Ravi *et al.*, 2009).

Drying of materials having high moisture content is a complicated unit operation process involving simultaneous, coupled heat and mass transfer, particularly under transient conditions (Diamante *et al.*, 2010; Karimi *et al.*, 2012). It is one of the oldest methods of preservation and one of the most important in post-harvest processing of fruits, vegetables and other agricultural products (Guine *et al.*, 2009; Mujumdar and Law, 2010). Also, it brings about substantial reduction in weight and volume thereby minimizing packaging, storage and transportation costs (Okos *et al.*, 1992). Drying not only affects the moisture content of the product, but also alters other physical, biological and chemical properties such as enzymatic activity, microbial spoilage, viscosity, hardness, aroma, flavor and palatability of foods (Ozbek and Dadali, 2007).

Many conventional thermal drying methods such as hot air drying result in slow drying rates in the falling rate period of drying (Zhang *et al.*, 2006). The long drying times at relatively high temperatures during the falling rate periods often lead to undesirable thermal degradation of the finished products and consume more energy and yield low drying efficiency (Alibas Ozkan *et al.*, 2007). Microwave drying is having advantage of high drying rates, high energy efficiency, better product quality and efficient space utilization. Microwave heating is a result of dipolar interaction of water molecules inside the food materials. The polar water molecule tend to align themselves according to change in electrical field and heat is produced due to friction between oscillating molecules. This rapid internal energy generation causes the pressure build up and results in rapid evaporation of water (Zang *et al.*, 2006; Dadali *et al.*, 2007a,b; Sutar and Prasad, 2007; Kumar *et al.*, 2011). Several studies have been carried out by researchers to investigate the microwave drying characteristics of agricultural materials. For example, Cabbage (Yanyang *et al.*, 2004), parsley (Soysal, 2004), Potato (Wang *et al.*, 2004), Mushroom (Rodriguez *et al.*, 2005), carrot (Wang and Xi, 2005), garlic (Sharma and Prasad, 2006), spinach (Alibas Ozkan *et al.*, 2007), okra (Dadali *et al.*, 2007a),

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Accepted: 9 January 2014
Received in revised form: 31 December 2013
Received: 8 November 2013

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pumpkin (Wang et al., 2007), Bamboo (Bal et al., 2010), basil (Demirhan and Ozbek, 2010), purslane (Demirhan and Ozbek, 2010), white mulberry (Evin, 2011), celery leaves (Demirhan and Ozbek, 2011), mango ginger (Krishna Murthy and Manohar, 2012). However, extensive literature survey shows there are no reports available on mathematical modeling and drying kinetics of elephant foot yam in spite of its high medicinal and food value.

Mathematical modeling of the drying process and equipment is an important aspect of drying technology in post-harvest processing of agricultural materials. Numerous mathematical equations can be found in literatures that describe drying phenomena of agricultural products. Among them, thin layer drying models have been found wide application due to their ease of use. These models can be categorized as theoretical, semi-theoretical and empirical model (Midilli et al., 2002; Park et al., 2002; Akpinar, 2006). The theoretical models suggest that the moisture transport is controlled mainly by internal moisture mechanism and needs assumption of geometry of the food material, mass diffusivity and heat conductivity (Babilis et al., 2006). Semi theoretical models and empirical models consider only the external resistance to the moisture transport and are valid only in the specific range drying conditions (Ozdemir and Devres, 1999). Semi-theoretical models are derived from the general solutions of Fick’s second law of diffusion. Empirical models derive a direct relationship between average moisture content and drying time. They neglect fundamentals of the drying process and their parameters have no physical meaning. Therefore, they cannot give clear and accurate view of the important processes occurring during drying although they may describe the drying curve for the conditions of the experiments (Hii et al., 2009).

The objective of present investigation is (a) to study the effect of microwave power on drying kinetics of elephant foot yam. (b) to select the best model among several thin layer drying models to describe the moisture removal behavior during microwave drying of elephant foot yam. (c) to compute effective moisture diffusivity and the activation energy.

**Materials and Methods**

*Experimental material*

Fresh Elephant foot yams were procured from the local market at Yeshwanthpur, Bangalore, India. Roots were washed in running tap water to remove adhered soil and stored in refrigerator at 4 ± 1°C until taken for further processing. Three 50 g of sliced elephant foot yam were dried in hot air oven (Neha scientific international, Model no. SI 101A) at 105°C for 24 hr to determine initial moisture content and expressed on dry basis (kg H₂O/kg db⁻¹). The average initial moisture content of the fresh elephant foot yam was 4.204 kg H₂O/kg db⁻¹. The elephant foot yams were peeled and cut into rectangular slices having the dimensions of 50 x 20 x 5 mm using sharp stainless steel knives. Only rectangular specimens having thickness with in ±0.5 mm difference of the specific dimensions were selected for drying process.

**Drying procedure**

Drying experiments were performed in domestic digital microwave oven (LG, India; Model MC-8087ABR). The microwave oven has the capability of operating at five different microwave powers, 180, 360, 540, 720 and 900 Watts. Microwave processing time and power level were adjusted with the digital control on the microwave oven. Entire experiment was carried out using sliced elephant foot yam of known weight of about 50 g and arranged as thin layer on the rotatable plate fitted inside the microwave oven cabin. The rotating plate provides the equal distribution of microwave radiation energy throughout the sample. Drying was performed at a single power level at a time. Weight loss was recorded at regular intervals of time using the digital weighing balance (CAS; Model MW-11-200 series) of accuracy 0.01 g. Microwave drying continued till the moisture reduces to about 0.01 kg H₂O/kg db⁻¹ of the initial moisture content. Drying experiments were conducted in triplicate and average values reported.

*Mathematical modeling of drying kinetics*

Experimental moisture content data of Elephant foot yam during microwave drying were converted to non-dimensionless moisture ratio using following equation:

\[ MR = \frac{X_t - X_e}{X_0 - X_e} \]  ... 1

Where \( X_0 \) is the initial moisture content, \( X_t \) is the moisture content at time \( t \) and \( X_e \) is the equilibrium moisture content (Ertekin and Yaldiz, 2004; Ozbek and Dadali, 2007). Above equation can be further simplified to MR = \( X_t/X_e \) as the values of \( X_e \) is relatively small compared to \( X_0 \) and \( X_t \) for long drying time (Diamante and Munro, 1993; Wang et al., 2007; Doymaz and Akgun, 2009).

The drying rate during the experiments was calculated using the following formula:

\[ \text{Rate of drying} = \frac{dX}{dt} = \frac{X_t - X_e}{X_0 - X_e} \]  ... 2
Where \(X_{t+dt}\) is the moisture content at time \(t+dt\) and \(X_t\) is the moisture content at time \(t\) is the drying time (Ozbek and Dadali, 2007). The experimental data of dimensionless moisture ratio Vs drying time were fitted to 10 different thin layer drying models widely used by many researchers Table 1.

**Calculation of effective moisture diffusivity**

Drying phenomenon of biological products takes place in the falling rate period after a short heating period (Ozbek and Dadali, 2007; Falade and Solademi, 2010). It is generally accepted that liquid diffusion is the only physical mechanism to transfer water to surface to be evaporated. Fick’s second law given below can be used to describe the drying process of the elephant foot yam. It is a unidirectional diffusion equation and can be used for various regularly shaped bodies such as rectangular, cylindrical and spherical products.

\[
\frac{\partial X}{\partial t} = D_{\text{eff}} \frac{\partial^2 X}{\partial z^2} \quad ... \ 3
\]

Where \(X\) is the moisture content (kg water kg db^{-1}), \(t\) is the time (s), \(z\) is the diffusion path (m), \(D_{\text{eff}}\) is the moisture dependent diffusivity (m^2/s) (Doymaz, 2004; Akpinar, 2006; Wang et al., 2007). Analytical solution to Fick’s second law was developed by Crank (1975) and following assumptions were made in arriving the solution: uniform distribution of initial moisture throughout the sample, negligible internal resistance to mass transfer, moisture transport/mass transfer by diffusion mechanism, Constant diffusion coefficient, negligible product shrinkage during drying, surface moisture content of the sample instantaneously reaches equilibrium with the condition of surrounding air (Doymaz, 2004; Ozbek and Dadali, 2007). Appropriate initial and boundary conditions for solving above equation are given below (Park et al., 2002; Akpinar, 2006):

\[
\begin{align*}
t &= 0, \ 0 < z < L, \ X = X_0 \\
t &> 0, \ z = 0, \ \frac{dX}{dt} = 0 \\
t &> 0, \ z = L, \ X = X_e
\end{align*}
\]

The solution of the equation for infinite slab of thickness of 2L is:

\[
\text{MR} = \frac{X_e - X_0}{X_0 - X_e} = \frac{8}{n^2} \sum_{n=0}^{\infty} \frac{(-1)^n \pi^2 D_{\text{eff}}}{4L^2} \exp \left( \frac{(2n+1)^2 \pi^2}{4L^2} t \right) \quad ... \ 4
\]

Eq. (5) could be further simplified to a straight line equation as given below:

\[
\ln(\text{MR}) = \ln \left( \frac{8}{n^2} \right) - \left( \frac{D_{\text{eff}} \pi^2}{4L^2} \right) t \quad ... \ 6
\]

Effective moisture diffusivity was typically determined by plotting experimental drying data in terms of \(\ln(\text{MR})\) Vs drying time and found from the slope \((\pi D_{\text{eff}} / 4L^2)\).

**Estimation of activation energy**

In the present investigation, temperature is not directly measurable quantity in the microwave oven used for drying. For the calculation of activation energy, modified form of Arrhenius equation as derived by Dadali et al. (2007a), illustrates the relationship between the kinetic rate constant and the ratio of the microwave power output to sample weight instead of temperature and the equation is described below:

\[
k = k_o \exp \left( \frac{-E_a}{MW} \right) \quad ... \ 7
\]

Where \(k_o\) (s^{-1}) is the pre-exponential factor, \(E_a\) is the activation energy (W g\(^{-1}\)), MW is the microwave power output (W), m is the mass of the sample (g).

**Statistical analysis**

Nonlinear Least square method using the SOLVER tool based on the Generalized Reduced Gradient (GRG) method of iteration available in Microsoft Excel (Microsoft Office 2010, USA) was used to fit the experimental data to selected models. For evaluating the goodness of fit, four statistical parameters such as residual sum square (RSS), root mean square error (RMSE), chi square (\(\chi^2\)), Relative percentage deviation (RPD) were used in addition to coefficient of determination (\(R^2\)) as primary criterion. The values of \(R^2\) were one of the primary criterions for selecting the best model and can be used to test linear relationship between experimental and model predicted values (Gunhan et al., 2005). High \(R^2\) (closer to 1) value represents the best fit. Statistical parameters may be computed from the following mathematical equations.

\[
R^2 = 1 - \frac{\Sigma_{i=1}^{N} (\text{MR}_{\text{pred,i}} - \text{MR}_{\text{exp,i}})^2}{\Sigma_{i=1}^{N} (\text{MR}_{\text{exp,i}} - \text{MR}_{\text{exp,i}})^2} \quad ... \ 9
\]

\[
\text{RSS} = \Sigma_{i=1}^{N} (\text{MR}_{\text{exp,i}} - \text{MR}_{\text{pred,i}})^2 \quad ... \ 10
\]
Where \( N \) is the total number of observations, \( p \) is number of factors in the mathematical model, \( \text{MR}_{\text{exp,i}} \) and \( \text{MR}_{\text{pre,i}} \) are (the experimental and predicted moisture ratio at any observation \( i \)). In nonlinear regression RSS is the important parameter and ideal value is zero (Sun and Byrne, 1998). RMSE and \( \chi^2 \) compare the differences between the model predicted values of moisture ratios to the experimental moisture ratios. The values of the RMSE and \( \chi^2 \) are always positive. Lower values indicate the closeness of experimental value with predicted value and goodness of fit (Midilli and Kucuk, 2003; Krishna Murthy and Manohar, 2013).

**Results and Discussions**

**Effect of microwave power and thickness on drying kinetics**

50 g elephant foot yam was dried at five microwave power output (900, 720, 540, 360, 180 W) in microwave dryer to study the effect of microwave power on moisture content, drying date and drying time, effective moisture diffusivity. The drying process was assumed to be finished when changes in moisture loss were negligible. Drying curves of moisture ratio vs. drying time reflecting the effect of microwave power is shown in Figure 1. Drying time to reduce the initial moisture content of 4.204 kg H\(_2\)O. kg db\(^{-1}\) to 0.01 kg H\(_2\)O.kg db\(^{-1}\) decreased significantly as the microwave power increases from 180 W to 900 W. When drying is carried out under different microwave power level at constant thickness the drying time required to reduce the moisture content to 0.01 kg H\(_2\)O.kg db\(^{-1}\) at 180 W is 4.5 to 5.6 times the time require at 900 W. inversely proportional to the microwave power.

Figure 2 shows the drying rate against the time at different drying conditions. Initially there is sudden increase in the drying rate and there is no constant drying rate period in the present investigation but a short accelerating period at the start. Same results were observed in the previous findings for different agricultural materials such as banana (Maskan, 2000), Parsley (Soyasal, 2004), Carrot (Wang and Xi, 2005), Lactose (Mcminn, 2006), apple pomace (Wang et al., 2007), bamboo (Bal et al., 2010), mango ginger (Krishna Murthy and Manohar, 2012) using microwave drying as reported by the authors.

Initially dehydration rate is very high, as seen from the Figure 3 to remove 50% of the moisture take 25-30% of the total drying time. Drying rate decreased rapidly with decrease in the moisture content and took more time to remove final moisture content. The average drying rates were in the range of 2.17 x 10\(^{-3}\) to 9.98 x 10\(^{-3}\) kg H\(_2\)O.Kg db\(^{-1}\).s\(^{-1}\) at selected experimental conditions. Drying rate was predominant in the falling rate period indicates that diffusion is the mechanism governing the moisture removal during the microwave drying of elephant foot yam.

**Evaluation of thin layer models**

The experimental dimensionless moisture ratio was regressed against the drying time according to the selected model presented in Table 1 of the ten thin layer drying models used to describe the effect of microwave power on drying kinetics, semi empirical.
Table 2. The estimated values of kinetic parameters of Midilli et al. model at different microwave power

<table>
<thead>
<tr>
<th>Microwave Power (MW), Watts</th>
<th>R²</th>
<th>RSS</th>
<th>RMSE</th>
<th>CS</th>
<th>k (min)</th>
<th>n</th>
<th>a</th>
<th>b</th>
</tr>
</thead>
<tbody>
<tr>
<td>180</td>
<td>0.9995</td>
<td>0.0009</td>
<td>0.0076</td>
<td>6.11E-05</td>
<td>0.0102</td>
<td>1.7424</td>
<td>0.9889</td>
<td>-1.40E-03</td>
</tr>
<tr>
<td>360</td>
<td>0.9998</td>
<td>0.0004</td>
<td>0.0052</td>
<td>3.58E-05</td>
<td>0.0402</td>
<td>1.7422</td>
<td>1.0027</td>
<td>7.91E-04</td>
</tr>
<tr>
<td>540</td>
<td>0.9989</td>
<td>0.0008</td>
<td>0.0118</td>
<td>4.15E-04</td>
<td>0.0629</td>
<td>1.9067</td>
<td>0.9931</td>
<td>9.63E-03</td>
</tr>
<tr>
<td>720</td>
<td>0.9998</td>
<td>0.0000</td>
<td>0.0011</td>
<td>3.81E-06</td>
<td>0.1535</td>
<td>1.7932</td>
<td>1.0005</td>
<td>-1.6E-02</td>
</tr>
<tr>
<td>900</td>
<td>0.9999</td>
<td>0.0001</td>
<td>0.0039</td>
<td>4.67E-05</td>
<td>0.1938</td>
<td>1.8526</td>
<td>0.9979</td>
<td>-2.11E-03</td>
</tr>
</tbody>
</table>

* R²-Coefficient of Determination, RSS-Residual Sum Square, RMSE-Root Mean Square Error, χ²-Chi Square, k- kinetic rate constant, a,b,n are model constants.

Table 3. Effective moisture diffusivities of elephant foot yam at different microwave power

<table>
<thead>
<tr>
<th>Microwave Power (MW), watts</th>
<th>R²</th>
<th>Effective Moisture Diffusivity (Dₑ, m²/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>180</td>
<td>0.9958</td>
<td>4.44E-09</td>
</tr>
<tr>
<td>360</td>
<td>0.9793</td>
<td>9.83E-09</td>
</tr>
<tr>
<td>540</td>
<td>0.9944</td>
<td>1.32E-08</td>
</tr>
<tr>
<td>720</td>
<td>0.9316</td>
<td>2.39E-08</td>
</tr>
<tr>
<td>900</td>
<td>0.9848</td>
<td>4.22E-08</td>
</tr>
</tbody>
</table>

Figure 3. Drying rate vs moisture content at different microwave powers

Midilli et al. (2003) model showed the good fit and regressed well with the experimental data, with high value of R² and low values of RSS, RMSE and χ². The drying kinetic parameters of Midilli et al. (2003) model were presented in Table 2. The drying rate constant obtained from Midilli et al. (2003) model was in the range of 0.0015 to 0.19 min⁻¹. Kinetic or drying rate constant was found to be increased with increase in microwave power. This implies that increase in the microwave power increases the moisture removal rate.

Effect of microwave power and thickness on effective moisture diffusivity

The method of slope was used to calculate the effective moisture diffusivities of elephant foot yam undergoing microwave drying. The plot of ln (MR) vs drying time gives a straight line with the slope (π²Dₑ/4L²) and values are presented in Table 3. Effective moisture diffusivity values of elephant foot yam under various drying conditions were estimated in the range of 4.44 x 10⁻⁹ m²/s to 4.2 x 10⁻⁸ m²/s. The values were within the range of 10⁻⁸ to 10⁻¹² m²/s for drying of food materials (Zogzas et al., 1996; Falade and Solademi, 2010). The Effective moisture diffusivity increased with increase in microwave power.

Estimation of activation energy

The kinetic rate constant (k) values regress well with ratio of sample mass to microwave power (m/MW) value. The coefficient of determination (R²), activation energy and pre exponential factor values are 0.96, 24.7 W/g⁻¹, 0.315 s⁻¹ respectively.

Conclusion

Elephant foot yam undergoing microwave drying was dried under different drying conditions to evaluate the effect of the microwave power on drying time, drying rate, drying rate constant, effective moisture diffusivity which helps in the simulation of drying process. Drying time was significantly decreased with increase in microwave power. Drying rate increased with increase microwave power. The average drying rate values were in the range of 2.17 x 10⁻³ to 9.98 x 10⁻¹ m/s H₂O.kg db⁻¹.s⁻¹ at microwave power level 180 to 900 W. A semi empirical Midilli et al. (2003) model showed a good fit among selected various empirical and semi empirical models to describe the drying kinetics. The kinetic rate constant obtained from the Midilli et al. (2003) model increased with increase in microwave power. The values of calculate effective moisture diffusivity using Fick’s diffusion for drying experiments at 180-900 W ranged from 4.44 x 10⁻⁹ m²/s to 4.2 x 10⁻⁸ m²/s. The drying studies on elephant foot yam will be of interest to applications involving drying of similar food materials like root vegetables. The modeling data can be used further for the optimization of the process parameters and to scale up drying process.

Acknowledgments

The authors are grateful to management and principal of Sapthagiri College of Engineering for providing research facilities. Authors would also like to extend their gratitude to Dr. Balaraman Manohar, Chief Scientist, CSIR-CFTRI, Mysore for his constant support and suggestions during the course of the study.
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